

Math 444 (Cowen)

Homework

Due 8 November 2010

**Text, page 135:** 1, 4 (DON'T use ' $\lim_{x \rightarrow \infty}$ '), 5, 6, 9, 10, 11

**Definition.** If  $I = [a, b]$  is a closed and bounded interval in  $\mathbb{R}$  and  $f$  is a real valued function defined on  $I$ , we say  $f$  is *strictly increasing on  $I$*  if for each  $x$  and  $y$  with  $a \leq x < y \leq b$ , we have  $f(x) < f(y)$ .

**A.** Let  $f$  be a strictly increasing, continuous function on the interval  $I = [a, b]$  and let  $J$  be the interval  $[f(a), f(b)]$ . Show that  $f$  is a bijection of  $I$  onto  $J$ .

**B.** Let  $f$  be a strictly increasing function defined on the interval  $I = [a, b]$  and let  $J = [f(a), f(b)]$ . Suppose, also, that  $f$  is a bijection of  $I$  onto  $J$ .

Prove that  $f$  is continuous on the interval  $I$ .