Math 444 (Cowen) **Homework** **Due 27 October 2010**

Text, page 104: 1, 4, 5, 9(ϵ-δ def), 10, 11ab, 12

From the test(!):

A. Let (a_n) be a convergent sequence of real numbers, say $\lim_{n\to\infty} a_n = \alpha$.

Define a sequence (u_k) by

$$u_k = \inf\{a_n : n \geq k\}$$

(a) Show that (u_k) is a bounded sequence.
(b) Show that (u_k) is an increasing sequence.
(c) Show that $\lim_{k\to\infty} u_k = \alpha$.

B. Define the sequence (y_n) inductively by $y_1 = 0$ and for each n in $\mathbb{N}$, $y_{n+1} = \dfrac{1}{3}(1 - y_n^2)$.
(a) Prove by induction that $0 \leq y_n \leq 1$ for each n in $\mathbb{N}$.
(b) Prove that the sequence (y_n) converges.
(c) Find $\lim_{n\to\infty} y_n$.