Math 35300 (Cowen) **First Class: Notes/Topics** **14 January 2014**

Review:

- Definitions of zero matrix, row, column, transpose, square, addition of matrices, product of matrices, identity, diagonal, invertible, [Hermitian, symmetric, self-adjoint], $\cdots$
- Determinants: some properties, ways of computing, some theorems
- Definitions of linear combination, span of a set of vectors, subspace, linearly independent set of vectors, basis, dimension, null space of a matrix, range of a matrix, rank-nullity theorem,
- Theorems connecting invertibility of a matrix with items above
- inner products (dot product)

Matlab Commands:

- matrix former: $[row; row;]$
- identity matrix: `eye(n)`
- zero matrix: `zeros(m,n)`
- conjugate transpose (adjoint) of A: `A'`
- inverse of A: `inv(A)`
- equation solver for $AX = b$: `X=A\b`
- rank of A: `rank(A)`
- determinant of A: `det(A)`
- basis for null space of A: `null(A)`
- basis for range of A: `orth(A)`

New Material:

- block matrices
- Norms of matrices(?)

Math 35300 (Cowen) **Handout 1** **14 January 2014**

For discussion Thursday, 16 January (among yourselves):
Quiz: Tuesday, January 21, 1:00p
on Complex Arithmetic and Review Material from Text (pages 30-192, below)

Please look over the material in chapters 1, 2 (except 2.4), 3 (sections 3.1 – 3.6), and 4 (sections 4.2 – 4.4), most of which is familiar to you. Be prepared to ask any questions you might have on this material. In addition, please do the following problems; they will not be collected on Tuesday, but you should ask questions about the ones you find difficult.

- page 30: 8, 9, 10, 12, 13, 14
- page 37: 1, 2, 3, 4, 5
- page 107: 9 (Note that the answer to problem 5 is $\det(E) = -142$.)
- page 167: 1, 2, 3, 4, 5
- page 192: 19

In addition, do the following problems:

A. Let A be a 20×20 matrix that has block form $A = \begin{pmatrix} B & C \\ 0 & D \end{pmatrix}$ where B is 4×15 and C is 4×5.

(a) What are the sizes of the 0 in the lower left corner and the matrix D?
(b) Show that the rank of A is less than or equal to 9.
(c) Show that the result of (b) is best possible by giving an example of such a matrix A that has rank exactly 9, that is, find matrices B, C, and D as above so that the resulting matrix A has rank exactly 9.

B. The vectors q_1, q_2, and q_3 are an orthogonal basis for the subspace $\mathcal{W}$ of $\mathbb{R}^4$, and satisfy $\|q_1\| = \sqrt{3}$, $\|q_2\| = \sqrt{2}$, $\|q_3\| = 2$.
In terms of this basis, vectors u, v, and w are expressed as

$$\begin{aligned} u &= q_1 - 2q_2 + q_3 \\ v &= 2q_1 + q_2 - 3q_3 \\ w &= 2q_1 + q_2 + 2q_3 \end{aligned}$$

(a) Find $\|u\|$.
(b) Find $< v, w >$.
(c) Show that the vectors u, v, and w are linearly independent.