

**Review:**

- Definitions of zero matrix, row, column, transpose, square, addition of matrices, product of matrices, identity, diagonal, invertible, [Hermitian, symmetric, self-adjoint],  $\dots$
- Determinants: some properties, ways of computing, some theorems
- Definitions of linear combination, span of a set of vectors, subspace, linearly independent set of vectors, basis, dimension, null space of a matrix, range of a matrix, rank-nullity theorem,
- Theorems connecting invertibility of a matrix with items above
- inner products (dot product)

**Matlab Commands:**

- matrix former: `[row; row; ]`
- identity matrix: `eye(n)`
- zero matrix: `zeros(m,n)`
- conjugate transpose (adjoint) of  $A$ :  $A'$
- inverse of  $A$ : `inv(A)`
- equation solver for  $AX = b$ : `X=A\b`
- rank of  $A$ : `rank(A)`
- determinant of  $A$ : `det(A)`
- basis for null space of  $A$ : `null(A)`
- basis for range of  $A$ : `orth(A)`

**New Material:**

- block matrices
- Norms of matrices(?)

**For discussion Thursday, 12 January:**

**Some to be handed in 17 January: page 167: 3, 5 page 192: 19 below: A, B**

Please look over the material in chapters 1, 2 (except 2.4), 3 (sections 3.1 – 3.6), and 4 (sections 4.2 – 4.4), most of which is familiar to you. Be prepared to ask any questions you might have on this material. In addition, please do the following problems; they will not be collected on Thursday, but you should ask questions about the ones you find difficult.

- page 37: 1, 2, 3, 4, 5
- page 107: 9 (Note that the answer to problem 5 is  $\det(E) = -142$ .)
- page 167: 1, 2, 3, 4, 5
- page 192: 19

In addition, do the following problems:

**A.** Let  $A$  be a  $20 \times 20$  matrix that has block form  $A = \begin{pmatrix} B & C \\ 0 & D \end{pmatrix}$  where  $B$  is  $4 \times 15$  and  $C$  is  $4 \times 5$ .

- What are the sizes of the 0 in the lower left corner and the matrix  $D$ ?
- Show that the rank of  $A$  is less than or equal to 9.
- Show that the result of (b) is best possible by giving an example of such a matrix  $A$  that has rank exactly 9, that is, find matrices  $B$ ,  $C$ , and  $D$  as above so that the resulting matrix  $A$  has rank exactly 9.

**B.** The vectors  $q_1$ ,  $q_2$ , and  $q_3$  are an orthogonal basis for the subspace  $\mathcal{W}$  of  $\mathbb{R}^4$ , and satisfy  $\|q_1\| = \sqrt{3}$ ,  $\|q_2\| = \sqrt{2}$ ,  $\|q_3\| = 2$ .

In terms of this basis, vectors  $u$ ,  $v$ , and  $w$  are expressed as

$$\begin{aligned} u &= q_1 - 2q_2 + q_3 \\ v &= 2q_1 + q_2 - 3q_3 \\ w &= 2q_1 + q_2 + 2q_3 \end{aligned}$$

- Find  $\|u\|$ .
- Find  $\langle v, w \rangle$ .
- Show that the vectors  $u$ ,  $v$ , and  $w$  are linearly independent.