Math 39000 (Cowen) **Handout 1** **11 January 2010**

For discussion Wednesday, 13 January:

Please look over the material in chapters 1, 2, 3 (sections 3.1 – 3.6), and 4 (sections 4.2 – 4.4), most of which is familiar to you. Be prepared to ask any questions you might have on this material. In addition, please do the following problems; they will not be collected, but you may wish to ask questions about the ones you find difficult.

- page 37: 1, 2, 3, 4, 5
- page 107: 9 (Note that the answer to problem 5 is $\det(E) = -142$.)
- page 167: 1, 2, 3, 4, 5
- page 192: 19

In addition, do the following problems:

A. Let A be a 20×20 matrix that has block form $A = \begin{pmatrix} B & C \\ 0 & D \end{pmatrix}$ where B is 4×15 and C is 4×5.

(a) What are the sizes of the 0 in the lower left corner and the matrix D?

(b) Show that the rank of A is less than or equal to 9.

(c) Show that the result of (b) is best possible by giving an example of such a matrix A that has rank exactly 9, that is, find matrices B, C, and D as above so that the resulting matrix A has rank exactly 9.

B. The vectors q_1, q_2, and q_3 are an orthogonal basis for the subspace $\mathcal{W}$ of $\mathbf{R}^4$, and satisfy $\|q_1\| = \sqrt{3}$, $\|q_2\| = \sqrt{2}$, $\|q_3\| = 2$.
In terms of this basis, vectors u, v, and w are expressed as

$$\begin{aligned} u &= q_1 - 2q_2 + q_3 \\ v &= 2q_1 + q_2 - 3q_3 \\ w &= 2q_1 + q_2 + 2q_3 \end{aligned}$$

(a) Find $\|u\|$.

(b) Find $<v, w>$.

(c) Show that the vectors u, v, and w are linearly independent.