Math 35100 (Cowen) **Due March 22**

Homework 8

1. In $\mathbf{R}^3$ with the usual inner product; for $v = (2, 1, -1)$ and $w = (1, -1, 1)$, find $\langle v, w \rangle$.
2. In $\mathbf{R}^4$ with the usual inner product; for $v = (1, 1, 2, -2)$ and $w = (2, 0, 1, 1)$, find $\langle v, w \rangle$.
3. In $\mathbf{R}^4$ with the usual inner product; for $v = (3, 0, 1, -1)$ and $w = (1, -2, 1, -1)$, find $\langle v, w \rangle$.

In each of the following, find the angle between v and w. (Use the usual inner product.)

4. $v = (3, 2, -1)$ and $w = (1, 0, -2)$.
5. $v = (2, -1, 2)$ and $w = (4, 4, -2)$.
6. $v = (1, -1, 2, 0)$ and $w = (3, -1, -1, 5)$.
7. Let $u = (1, -2, 1, 3)$ and $v = (2, 1, -2, 1)$. Find $\|u\|$, $\|v\|$, and $\|u + v\|$ and observe that $\|u + v\| \leq \|u\| + \|v\|$.
8. The Parallelogram Law from Euclidean Geometry is: The sum of the squares of the lengths of the diagonals is equal to the sum of the squares of the lengths of the sides. If u and v are vectors that form the sides of a parallelogram, then the diagonals are $u + v$ and $u - v$. Prove the vector form of the Parallelogram Law
$$\|u + v\|^2 + \|u - v\|^2 = 2\left(\|u\|^2 + \|v\|^2\right)$$