Homework 5

1. Find the inverse of $\begin{pmatrix} 1 & -1 & 3 \\ 4 & 0 & -1 \\ -3 & 1 & 2 \end{pmatrix}$ by using determinants and the "Adjugate Formula".

2. Use Cramer's Rule to solve the system $\begin{cases} -y + z = -4 \\ -x + 3y + 3z = 4 \\ 2x + 3z = 1 \end{cases}$

3. Let $\mathcal{W} = \{(s, 0, t, 2s + 3t) : s, t \text{ real}\}$. Show that $\mathcal{W}$ is a subspace of $\mathbb{R}^4$.

4. Let $\mathcal{U} = \{(s, 3, t, 2s + 3t) : s, t \text{ real}\}$. Show that $\mathcal{U}$ is not a subspace of $\mathbb{R}^4$.

5. Show that the system
$$\begin{cases} w + 2x - y + z = 2 \\ 3x - 2y + 2z = 4 \\ 2w - x + y - z = -1 \end{cases}$$
has infinitely many solutions but that the set of solutions of this system does *not* form a subspace of $\mathbb{R}^4$.

6. Show that the set $\mathcal{W}$ of Exercise 3 is the range of the matrix
$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 2 & 3 \end{pmatrix}$$
which means that $\mathcal{W}$ is a subspace by the Example from class.

7. Let $\mathcal{Z} = \{(2s + t, s, -s + 3t, t) : s, t \text{ real}\}$. Find a matrix A as in Exercise 6 to show that $\mathcal{Z}$ is a subspace of $\mathbb{R}^4$ by the Example from class.

8. Let $\mathcal{V} = \{(s - t, -s + 3t, s, t) : s, t \text{ real}\}$. Show that $\mathcal{V}$ is a subspace of $\mathbb{R}^4$ by showing it is the nullspace of the matrix
$$A = \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & -3 \end{pmatrix}$$
and using the result of the Example from class.

9. Write $(1, -1, 5, -5)$ as a linear combination of the vectors $(1, 1, -1, 1)$, and $(2, 1, 1, -1)$ or explain why it is not possible to do so.

10. Write $(1, -1, 1, 2)$ as a linear combination of the vectors $(1, 1, -1, 1)$, and $(2, 1, 1, -1)$ or explain why it is not possible to do so.

11. Find vectors that span $\mathcal{N}(B)$ the nullspace of the matrix $B = \begin{pmatrix} 1 & 1 & 1 & -2 \\ 2 & 3 & 0 & -1 \end{pmatrix}$

12. Find vectors that span $\mathcal{N}(C)$ the nullspace of the matrix $C = \begin{pmatrix} 1 & -1 & 0 & 2 \\ 2 & 1 & 1 & 0 \\ 3 & 0 & 1 & 2 \end{pmatrix}$

13. Find vectors that span $\mathcal{R}(C)$, the range of the matrix $C = \begin{pmatrix} 1 & -1 & 0 & 2 \\ 2 & 1 & 1 & 0 \\ 3 & 0 & 1 & 2 \end{pmatrix}$