**Homework 4**

1. Let $A = \begin{pmatrix} 4 & 3 & -2 \\ 2 & -5 & 6 \end{pmatrix}$ and let $B = \begin{pmatrix} 0 & -1 & 3 \\ 2 & -1 & 6 \\ 5 & 2 & 1 \end{pmatrix}$

   (a) Find $AB$ (from the definition) as a $2 \times 3$ matrix.

   (b) Partition $A$ as $\left( A_{11} \mid A_{12} \right)$ and $B$ as $\left( \begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array} \right)$, where both $A_{11}$ and $B_{11}$ are $2 \times 2$ matrices, that is, say what each of $A_{11}, A_{12}, \cdots, B_{22}$ are.

   (c) Determine each of the relevant products from (b) above and find $AB$ as a partitioned matrix.

2. Explore how your software handles block matrices.

   (a) Enter the matrices

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 \\ 3 & 1 \end{pmatrix}$$
$$C = \begin{pmatrix} 4 & 2.6 & 0 \\ 3 & -.3 & 8 \end{pmatrix}, \quad D = \begin{pmatrix} 3 & -2 \\ 1.5 & 4 \end{pmatrix}$$

   (b) Make a $4 \times 5$ matrix $E$ from the matrices $A$, $B$, $C$, and $D$ to get

$$E = \left( \begin{array}{rrr|rr} 1 & 2 & -1 & 1 & -1 \\ 0 & -1 & 4 & 3 & 1 \\ \hline 4 & 2.6 & 0 & 3 & -2 \\ 3 & -.3 & 8 & 1.5 & 4 \end{array} \right)$$

   You probably do not need to retype all the entries! Note that $E = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$

   (c) Make a $4 \times 4$ matrix $F$ from $E$ by deleting the last column of $E$.

3. Show that if $A$ is an $m \times n$ matrix and $B$ is an $n \times p$ matrix whose $k^{th}$ column is zero, then the $k^{th}$ column of $AB$ is zero.

4. Suppose $A$ is a square matrix partitioned as

$$A = \left( \begin{array}{c|c} X & Y \\ \hline 0 & Z \end{array} \right)$$

   where $X$ and $Z$ are square invertible matrices and 0 is a zero matrix.

   (a) Find formulas for $P$, $Q$, $R$, and $S$ so that the block matrix

$$\left( \begin{array}{c|c} P & Q \\ \hline R & S \end{array} \right)$$

   is $A^{-1}$. (*Caution:* matrix multiplication is not commutative!) If you are successful, you will have shown that matrices with the given block form are invertible.

   (b) Use your formula to find $A^{-1}$ when $X = \left( -1 \right)$, $Y = \left( 1 \quad -1 \right)$, and $Z = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ (Note that $Z^{-1} = \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$ )

Find the determinants of the following matrices.

5. $A = \begin{pmatrix} -2 & 0 & 1 \\ 3 & 4 & -2 \\ 5 & 3 & -1 \end{pmatrix}$

6. $B = \begin{pmatrix} 1 & 2 & 0 & 1 \\ -2 & -2 & -1 & 1 \\ 2 & 2 & 4 & 0 \\ 3 & 6 & 3 & 3 \end{pmatrix}$

7. $C = \begin{pmatrix} 1 & 1 & -2 & 1 \\ -1 & 4 & 3 & -2 \\ 0 & 5 & 3 & 0 \\ -2 & 3 & 7 & 1 \end{pmatrix}$

8. $D = \begin{pmatrix} 0 & 0 & 3 & -1 \\ 1 & 2 & -1 & 1 \\ -2 & -2 & 3 & -5 \\ 1 & 6 & -8 & -4 \end{pmatrix}$

9. $E = \begin{pmatrix} -1 & 3 & -4 & 2 \\ 0 & -1 & 1 & 3 \\ 3 & 0 & -2 & -1 \\ 2 & -1 & 4 & 3 \end{pmatrix}$

10. $F = \begin{pmatrix} 0 & 0 & -4 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 \\ 0 & -2 & -1 & 4 & -3 \\ 2 & -1 & 4 & 3 & 0 \\ -2 & 3 & -9 & -11 & 10 \end{pmatrix}$