## Homework 2

Solve the following systems.

1. $$\begin{cases} 2w - x + 2y + z &= 1 \\ x + y - z &= -2 \\ 3y + z &= 0 \\ 2z &= 6 \end{cases}$$

2. $$\begin{cases} 2w - x + 2y + z &= 0 \\ x + y - z &= 0 \\ y - z &= 0 \end{cases}$$

(Hint: solve for $w$, $x$, and $y$ in terms of $z$. There will be infinitely many solutions, one for each value of $z$.)

Use your software to solve the following systems. Be sure to check your answers!

3. $$\begin{cases} x - y + z &= 1 \\ -x + 3y + 3z &= 5 \\ 2x + 3z &= 4 \end{cases}$$

4. $$\begin{cases} w + 2x - y + 3z &= 1 \\ 3w + x + 2y + 4z &= 8 \\ -x + y - z &= 1 \end{cases}$$

5. $$\begin{cases} w + 2x - y + 3z &= 1 \\ 3w + x + 2y + 4z &= 1 \\ -x + y - z &= 2 \end{cases}$$

6. Consider the system:

$$\begin{cases} u + 2v + w - x - 2y &= 3 \\ -2u + v + w + x + 2y &= 5 \\ u + v - w + 2x + 4y &= -2 \\ u - v + 3x + y &= -7 \\ -u + 3v + w + x + 3y &= 7 \end{cases}$$

   (a) Choose $A$ and $b$ so that the system can be written in matrix form as $AX = b$ where $X = (u, v, w, x, y)$.
   (b) Check that $X_p = (-1, 1, 2, -2, 1)$ is a solution of the system and check that $X_0 = (-1, 1, -2, 1, -1)$ is a solution of the associated homogeneous system $AX = 0$.
   (c) Without using Gaussian elimination or a machine, find two other non-trivial solutions of $AX = 0$.
   (d) Without using Gaussian elimination or a machine, find two other solutions of $AX = b$.

7. The five-tuples $(2, 2, 1, -1, 1)$ and $(1, 1, 2, -1, -1)$ are both solutions of the system:

$$\begin{cases} a + b + 4c + d + e &= 8 \\ a - b + 2c + 2d + e &= 1 \\ 2a + b - c - d - 2e &= 4 \\ b + 3c + d + e &= 5 \\ 2a - b + c + 3d &= 0 \end{cases}$$

   (a) Without using Gaussian elimination or a machine, write down two non-trivial solutions of the associated homogeneous system.
   (b) Write down two other solutions of the given system.

8. Let $A$ be the matrix

$$\begin{pmatrix} 1 & -1 & 2 & 1 \\ 2 & 1 & -3 & -1 \\ 1 & 1 & 3 & -2 \\ -1 & 2 & -2 & 3 \end{pmatrix}$$

and let $b = (3, -1, 3, 2)$ and let $c = (0, 4, -4, 4)$.

(a) Check that $Y = (1, 1, 1, 1)$ solves the system $AX = b$ and that $Z = (1, 0, -1, 1)$ solves the system $AX = c$.

(b) Without using Gaussian Elimination or a machine, find a solution of the system $AX = (6, -2, 6, 4) = 2b$.

(c) Without using Gaussian Elimination or a machine, find a solution of the system $AX = (3, 3, -1, 6) = b + c$.

(d) Without using Gaussian Elimination or a machine, find a solution of the system $AX = (9, 5, 1, 14) = 3b + 2c$.