Homework 14

1. A is a 3×3 matrix;
   the vector u is an eigenvector of A with eigenvalue -1;
   the vector v is an eigenvector of A with eigenvalue 2;
   and the vector w is an eigenvector of A with eigenvalue 5.
   (a) Evaluate: $(A^2 + 4A - I)(u)$.
   (b) Evaluate: $(A^2 + 4A - I)(v)$.
   (c) Evaluate: $(A^2 + 4A - I)(w)$.
   (d) Evaluate: $(A^2 + 4A - I)(3u - 2v + w)$.

2. Let B be an $n \times n$ matrix whose eigenvalues are 1, 2, -3, and 3.
   Find the four eigenvalues of $B^2 - B + 3I$.

3. Let C be an $n \times n$ matrix that is invertible, let λ be an eigenvalue of C, and let x be an eigenvector of C corresponding to λ. Show that $\lambda \neq 0$ and that λ^{-1} is an eigenvalue of C^{-1} with eigenvector x.

4. If w is an eigenvector of B (of Exercise 2 above) corresponding to the eigenvalue 2, find $(B - 5I)^{-1}w$.

5. Suppose N is an $n \times n$ matrix such that $N^k = 0$ for some positive integer k.
   Find the eigenvalues of N.

6. Suppose P is a matrix such that $P = P^2$. Find the eigenvalues of P.

7. Find an invertible matrix S and a diagonal matrix D so that $D = S^{-1}RS$ where

$$R = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$$

8. Find an invertible matrix S and a diagonal matrix D so that $D = S^{-1}AS$ where

$$A = \begin{pmatrix} -6 & -4 & 1 \\ 6 & 5 & 0 \\ -8 & -4 & 3 \end{pmatrix}$$

9. Find an invertible matrix S and a diagonal matrix D so that $D = S^{-1}BS$ where

$$B = \begin{pmatrix} 5 & -4 & 4 \\ 2 & -1 & 2 \\ -1 & 1 & 0 \end{pmatrix}$$

10. Do the necessary calculations, then explain why it is not possible to diagonalize the matrix

$$J = \begin{pmatrix} 0 & -3 & -7 \\ 1 & 5 & 10 \\ -1 & -2 & -3 \end{pmatrix}$$