Homework 12

1.
$$\begin{cases} x + 2y = 3 \\ 2x + 3y = -1 \\ 2x + 2y = 1 \end{cases}$$
   (a) Explain, in terms of ranges of matrices, why this system is inconsistent.
   (b) Find the least squares solution to the system.
   (c) Explain the relationship of your answer to (b) above and ranges of matrices.

Use the normal equations to find the "best" (i.e. least squares) solution to the following overdetermined systems. Decide whether each system is consistent or inconsistent. If you are using MATLAB or other software that gives the least squares solution directly, compare your answers to the solution given by your software.

2. $\begin{cases} 2x - 3y = 1 \\ -x + 2y = 3 \\ -x + y = -1 \end{cases}$

3. $\begin{cases} a + b = 2 \\ 2a + b = 4 \\ a + 3b = 0 \\ 2a - b = 6 \end{cases}$

4. $\begin{cases} r - 2s = 3 \\ -r + 3s = 2 \\ r - 2s = -1 \end{cases}$

5. $\begin{cases} r - 2s + 3t = -1 \\ 2r + s - t = 2 \\ s + 2t = 0 \\ -r + 2s + t = 3 \end{cases}$

6. A thermocouple is supposed to put out a voltage in response to a temperature so that the graph of temperature versus voltage is a straight line. In calibrating a thermocouple for their ChemE lab, Mary and John obtain the data $p_1 = (5, -8)$; $p_2 = (13, -6)$; $p_3 = (22, -3)$; and $p_4 = (39, 0)$ where the first coordinate is the output voltage in microvolts and the second coordinate is the temperature in degrees Celsius. Find the line that best fits this data (i.e., the least squares regression line).

7. A large meteorite is following an orbit that is nearly parabolic with the sun at the focus of the parabola. Measurements have been made which have very accurately determined the direction of the axis, and it is desired to predict the future course of the meteorite to determine if any significant collisions are likely. In a suitable coordinate system, the path of the meteorite should be $d = \alpha + \beta t + \gamma t^2$ for some choice of α, β, and γ. The four measurements that are to be used in the prediction are $d = -1.6$ when $t = -2$; $d = 2.1$ when $t = -1$; $d = 3.4$ when $t = 0$; and $d = 1.8$ when $t = 1$. Find an optimal choice (i.e. least squares) for the values of α, β, and γ and use them to predict the value of d for $t = 5$.

8. A simple model for the air resistance of a moving body is $R = av + bv^2$ where R is the force due to air resistance, v is the velocity of the object, and a and b are constants that depend on the shape of the object.
   (a) Find the best (least squares) estimates for a and b based on the data below:

Air Resistance Data

Force	Velocity
2	1
7	2
18	5
45	10
80	20
144	30

   (b) Use the parameters you have computed to estimate the force due to the air resistance of the object whose velocity is 40.