Homework 9

You should use **Matlab** or **Octave** to do the following problems.

- It is **NOT** acceptable to hand in printouts!!
- For decimal approximations, it is enough to give 2 decimal places in your answers.
- When using a machine, explain what it did.

1. The following system has infinitely many solutions:
$$\begin{cases} \phantom{5v + {}} w + 2x + 3y + 4z = -4 \\ 5v + 6w + 7x + 8y + 9z = 1 \\ v + w + x + y + z = 1 \\ v + x + z = 1 \end{cases}$$
Find the general solution and *find three solutions* explicitly.

2. Let $w_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix}$, $w_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $w_3 = \begin{pmatrix} 1 \\ -5 \\ 3 \\ -1 \end{pmatrix}$, $w_4 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}$, and $w_5 = \begin{pmatrix} -2 \\ 1 \\ 1 \\ -1 \end{pmatrix}$.
   (a) Explain how you can tell that $\{w_1, w_2, w_3, w_4, w_5\}$ is a linearly dependent set without doing any calculations.
   (b) Write one of the vectors as a linear combination of the rest.

3. Let $p_1 = (1, -1, 1, 1, 1)$, let $p_2 = (1, 1, 0, -2, 2)$, and let $q = (5, 1, 2, -4, 8)$.
   (a) Write q as a linear combination of p_1 and p_2.
   (b) Let $r = (8, -2, -4, 5, 1)$. Write r as a linear combination of p_1, p_2, and q or show that it is not a linear combination of them.
   (c) What is the dimension of the subspace spanned by p_1, p_2, q, and r? Explain your answer!

4. Let $\mathcal{W}$ be the column space of F, the subspace spanned by the columns of F, where
$$F = \begin{pmatrix} 2 & 1 & 1 & 1 \\ -1 & -1 & -2 & 1 \\ 3 & 1 & 0 & -2 \\ 1 & 1 & 2 & 1 \end{pmatrix}$$
   (a) Choose some of the column vectors of F to get a basis for $\mathcal{W}$ and explain why your answer is correct.
   (b) If $\mathcal{W} = \mathbb{R}^4$, explain why this is true.
   If $\mathcal{W}$ is *NOT* $\mathbb{R}^4$, find a vector x in $\mathbb{R}^4$ that is *NOT* in $\mathcal{W}$ and explain why it is not.

5. Suppose the vectors $\{u_1, u_2, u_3, u_4\}$ are a basis for $\mathbb{R}^4$.
Show that if $y_1 = 2u_1 - 3u_4$, $y_2 = u_1 + 2u_2 - u_3$, $y_3 = u_2 + 2u_3 - u_4$ and $y_4 = u_1 + 2u_2 - u_3 + u_4$, then $\{y_1, y_2, y_3, y_4\}$ is also a basis for $\mathbb{R}^4$.

6.
$$\text{Let } E = \begin{pmatrix} 2 & 1 & 0 & 1 & -2 \\ 0 & 1 & 1 & 2 & 3 \\ -1 & 1 & 1 & -2 & 1 \\ 1 & 3 & 2 & 1 & 2 \end{pmatrix}$$
Find bases for $\mathcal{N}(E)$ the nullspace of E and $\mathcal{R}(E)$ the range of E.
In addition, find some columns of E that form a basis for $\mathcal{R}(E)$.