Homework 3

Solve the following systems by Gaussian elimination. If there is only one solution, say that the solution is unique; if there is no solution, say so; if there are infinitely many, say so **and** find the general solution **and** find three explicit solutions. (You should check your answers by substitution or using MATLAB!)

1. $\begin{cases} x - 2y &= 2 \\ -2x - 4y &= -3 \end{cases}$

2. $\begin{cases} 2x - y &= 3 \\ -4x + 2y &= -6 \end{cases}$

3. $\begin{cases} 3x - 2y &= 7 \end{cases}$

4. $\begin{cases} 2x + y &= 4 \\ 3x - y &= 11 \\ x - 2y &= 7 \end{cases}$

5. $\begin{cases} 2x + y &= 3 \\ 3x - y &= -1 \\ x - 2y &= 2 \end{cases}$

6. $\begin{cases} x + y - z &= 4 \\ -x + y - 2z &= -3 \\ 2x - 4y + 8z &= 6 \end{cases}$

7. $\begin{cases} 3x + 2y + z &= 4 \\ 2x + y + 2z &= -2 \end{cases}$

8. $\begin{cases} x - y - 2z &= 3 \\ 2x + y + z &= 3 \\ -5x + y + 3z &= -11 \\ x + 2y - z &= 0 \end{cases}$

9. $\begin{cases} x - y - 2z &= 2 \\ 2x + y + z &= 1 \\ -5x + y + 3z &= 7 \\ x + 2y - z &= 0 \end{cases}$

10. $\begin{cases} x - 3y + 2z &= 6 \end{cases}$

11. $\begin{cases} w - x + y + 2z &= -1 \\ -2w + 3x + y - 3z &= 2 \\ w + 2x + 9y + 7z &= -3 \\ 2w - x + 6y + 4z &= -1 \\ w - 2x - 2z &= 2 \end{cases}$

12. Letting $X = (a, b, c, d, e)$,

$$X = \begin{pmatrix} -2 \\ 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

is a solution of the system

$$\begin{cases} a + b - c - d + 4e &= 5 \\ 2a + b + c \phantom{{}-d} + 3e &= -1 \\ -a + 3b + c + 5d + 4e &= 3 \end{cases}$$

for every s and t and every solution can be written in this form for some s and t.

(a) Show that $X = (1, 4, -4, 0, -1)$ is a solution of the system and find values of s and t that correspond to this solution.

(b) Without using Gaussian elimination or a machine, find two non-trivial solutions of the system $AX = 0$.

(c) $X = (2, -1, 0, 1, 3)$ is a solution of the system $AX = (12, 12, 12)$. Without using Gaussian elimination or a machine, find all solutions of the system $AX = (12, 12, 12)$.

Find the inverses of the following matrices. (You should check your answers!)

13. $\begin{pmatrix} 1 & 0 \\ 3 & -1 \end{pmatrix}$

14. $\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}$

15. $\begin{pmatrix} 1 & 2 & -1 & -3 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

16. $\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$

17. $\begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix}$

18. $\begin{pmatrix} 1 & -1 & 2 \\ 2 & -3 & 3 \\ 3 & -2 & 5 \end{pmatrix}$

19. $\begin{pmatrix} 1 & 1 & -1 \\ -1 & 0 & 4 \\ 2 & 3 & 1 \end{pmatrix}$

20. If A is an $m \times n$ matrix, we say B is a *left inverse* of A if $BA = I$ and we say C is a *right inverse* of A if $AC = I$. Observing that identity matrices are square, consideration of sizes shows that if B or C exist, they must be $n \times m$ matrices.
   (a) Find left and right inverses (or say if they do not exist) for the matrix
   $$A = \begin{pmatrix} 1 & -1 \\ 1 & -2 \\ 2 & -3 \end{pmatrix}$$
   (b) Find left and right inverses (or say if they do not exist) for the matrix
   $$E = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & -2 \end{pmatrix}$$
   (c) Can you propose a general statement about when left and right inverses of an $m \times n$ matrix exist?