Homework 2

1. (a) Let $A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 2 \\ 2 & -1 & 1 \end{pmatrix}$ and let $B = \begin{pmatrix} -4 & -1 & 2 \\ -5 & -1 & 2 \\ 3 & 1 & -1 \end{pmatrix}$.
   Explain why $A = B^{-1}$.
   (b) Is $B = A^{-1}$? Explain!
   (c) Let $C = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$, and $D = \begin{pmatrix} -2 & 1 \\ -3 & 1 \\ 1 & 1 \end{pmatrix}$. Is $D = C^{-1}$? Explain!

2. Verify that if N is a matrix such that $N^4 = 0$, then

$$(I - N)^{-1} = I + N + N^2 + N^3.$$

   WARNING! Such matrices are called *nilpotent* and are **not** necessarily 0.
   For example, the matrix $M = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$ satisfies $M^2 = 0$.

3. Let E be an $m \times n$ matrix.
   (a) Show that EE' and $E'E$ are both Hermitian.
   (b) Give an example to show that these are not always the same.

4. Redo Exercise 7 from Homework 1 and Exercise 1 above using a suitable machine. How does your machine react to undefined matrix operations?

5.

$$F = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 4 & 3 \\ 4 & 2.6 & 0 & 3 \\ 3 & -.3 & 8 & 1.5 \end{pmatrix}$$

   (a) Use a suitable machine to find $G = F^{-1}$
   (b) Find the computed values of GF and $GF - I$. Explain the output of your machine.

Solve the following systems.

6. $\begin{cases} 2w - x + 2y + z = 1 \\ x + y - z = -2 \\ 3y + z = 0 \\ 2z = 6 \end{cases}$

7. $\begin{cases} 2w - x + 2y + z = 0 \\ x + y - z = 0 \\ y - z = 0 \end{cases}$

   (Hint: solve for w, x, and y in terms of z. There will be infinitely many solutions, one for each value of z.)

Use your software to solve the following systems. Be sure to check your answers!

8. $\begin{cases} x - y + z = 1 \\ -x + 3y + 3z = 5 \\ 2x + 3z = 4 \end{cases}$

9. $\begin{cases} w + 2x - y + 3z = 1 \\ 3w + x + 2y + 4z = 8 \\ -x + y - z = 1 \end{cases}$

10. $\begin{cases} w + 2x - y + 3z = 1 \\ 3w + x + 2y + 4z = 1 \\ -x + y - z = 2 \end{cases}$

11. Consider the system:

$$\begin{cases} u+2v+w-x-2y &= 3 \\ -2u+v+w+x+2y &= 5 \\ u+v-w+2x+4y &= -2 \\ u-v \qquad +3x+y &= -7 \\ -u+3v+w+x+3y &= 7 \end{cases}$$

   (a) Choose A and b so that the system can be written in matrix form as $AX = b$ where $X = (u, v, w, x, y)$.
   (b) Check that $X_p = (-1, 1, 2, -2, 1)$ is a solution of the system and check that $X_0 = (-1, 1, -2, 1, -1)$ is a solution of the associated homogeneous system $AX = 0$.
   (c) Without using Gaussian elimination or a machine, find two other non-trivial solutions of $AX = 0$.
   (d) Without using Gaussian elimination or a machine, find two other solutions of $AX = b$.

12. The five-tuples $(2, 2, 1, -1, 1)$ and $(1, 1, 2, -1, -1)$ are both solutions of the system:

$$\begin{cases} a+b+4c+d+e &= 8 \\ a-b+2c+2d+e &= 1 \\ 2a+b-c-d-2e &= 4 \\ b+3c+d+e &= 5 \\ 2a-b+c+3d &= 0 \end{cases}$$

   (a) Without using Gaussian elimination or a machine, write down two non-trivial solutions of the associated homogeneous system.
   (b) Write down two other solutions of the given system.

13. Let A be the matrix

$$\begin{pmatrix} 1 & -1 & 2 & 1 \\ 2 & 1 & -3 & -1 \\ 1 & 1 & 3 & -2 \\ -1 & 2 & -2 & 3 \end{pmatrix}$$

and let $b = (3, -1, 3, 2)$ and let $c = (0, 4, -4, 4)$.
   (a) Check that $Y = (1, 1, 1, 1)$ solves the system $AX = b$ and that $Z = (1, 0, -1, 1)$ solves the system $AX = c$.
   (b) Without using Gaussian Elimination or a machine, find a solution of the system $AX = (6, -2, 6, 4) = 2b$.
   (c) Without using Gaussian Elimination or a machine, find a solution of the system $AX = (3, 3, -1, 6) = b + c$.
   (d) Without using Gaussian Elimination or a machine, find a solution of the system $AX = (9, 5, 1, 14) = 3b + 2c$.