Homework 13

1. Let $u = (1,1,1,1,1)$, $v = (0,1,0,1,0)$, $w = (1,0,1,0,1)$, $x = (1,1,0,0,0)$, and $y = (0,0,1,1,1)$. Some of these vectors are eigenvectors for the matrix
$$E = \begin{pmatrix} 6 & -3 & -1 & 4 & -2 \\ 1 & 2 & -1 & 2 & 0 \\ 3 & -3 & 2 & 4 & -2 \\ 1 & -1 & -1 & 5 & 0 \\ 2 & -2 & -1 & 3 & 2 \end{pmatrix}$$
For each vector, decide if it is an eigenvector of E or not, and if it is, find the corresponding eigenvalue.

2. The eigenvalues of
$$P = \begin{pmatrix} 2 & 4 \\ 1 & -1 \end{pmatrix}$$
are 3 and -2. Find an eigenvector for each eigenvalue of P.

3. The eigenvalues of
$$Q = \begin{pmatrix} -4 & -12 & -6 \\ 3 & 8 & 3 \\ -3 & -6 & -1 \end{pmatrix}$$
are -1 and 2. Find a basis for each eigenspace of Q.

Find the eigenvalues and bases for the eigenspaces for each of the following matrices.

4. $\begin{pmatrix} 2 & 3 \\ -1 & 6 \end{pmatrix}$

5. $\begin{pmatrix} 8 & -10 \\ 5 & -7 \end{pmatrix}$

6. $\begin{pmatrix} 0 & 2 & -4 \\ 2 & -3 & -2 \\ -4 & -2 & 0 \end{pmatrix}$

7. $\begin{pmatrix} -8 & 14 & 22 \\ -4 & 4 & 8 \\ -1 & 4 & 5 \end{pmatrix}$

8. $\begin{pmatrix} 2 & 1 & 0 \\ -1 & 1 & 1 \\ -3 & -4 & 0 \end{pmatrix}$

9. $\begin{pmatrix} 2 & -5 & -4 \\ -2 & 3 & 3 \\ 4 & -8 & -7 \end{pmatrix}$

10. Find the eigenvalues and bases for the eigenspaces of the matrix
$$\begin{pmatrix} 36 & -24 & 3 & -11 & 4 \\ 49 & -34 & 1 & -18 & 6 \\ -25 & 15 & -4 & 5 & -1 \\ 13 & -6 & 5 & 1 & -1 \\ 37 & -24 & 1 & -12 & 2 \end{pmatrix}$$
Be careful to interpret the answers given by your machine correctly!

11. (a) Prove that if E is an $n \times n$ matrix, then the eigenvalues of E^t, the transpose of E, are the same as those of E.
    (b) Give an example of a 2×2 matrix, E, such that the eigenvectors of E^t are different from those of E.