Math 35100 (Cowen) **Due November 2**

Homework 10

1. In $\mathbf{R}^3$ with the usual inner product; $v = (2, 1, -1)$ and $w = (1, -1, 1)$.
2. In $\mathbf{R}^4$ with the usual inner product; $v = (1, 1, 2, -2)$ and $w = (2, 0, 1, 1)$.
3. In $\mathbf{R}^4$ with the usual inner product; $v = (3, 0, 1, -1)$ and $w = (1, -2, 1, -1)$.

In each of the following, find the angle between v and w. (Use the usual inner product.)

4. $v = (3, 2, -1)$ and $w = (1, 0, -2)$.
5. $v = (2, -1, 2)$ and $w = (4, 4, -2)$.
6. $v = (1, -1, 2, 0)$ and $w = (3, -1, -1, 5)$.
7. Let $u = (1, -2, 1, 3)$ and $v = (2, 1, -2, 1)$. Find $\|u\|$, $\|v\|$, and $\|u + v\|$ and observe that $\|u + v\| \leq \|u\| + \|v\|$.
8. The Parallelogram Law from Euclidean Geometry is: The sum of the squares of the lengths of the diagonals is equal to the sum of the squares of the lengths of the sides. If u and v are vectors that form the sides of a parallelogram, then the diagonals are $u + v$ and $u - v$. Prove the vector form of the Parallelogram Law
$$\|u + v\|^2 + \|u - v\|^2 = 2\left(\|u\|^2 + \|v\|^2\right)$$
9. (a) Use the Theorem on orthogonal sets to show that the vectors $w_1 = (1, 1, 0)$, $w_2 = (1, -1, 1)$, and $w_3 = (-1, 1, 2)$ are a basis for $\mathbb{R}^3$.
   (b) Use the corresponding expansion theorem to write $v = (2, -1, 3)$ as a linear combination of w_1, w_2, and w_3.
10. The vectors u_1, u_2, u_3, u_4 are orthogonal vectors that span the subspace $\mathcal{U}$ of $\mathbb{R}^{11}$.
    Moreover, $\|u_1\| = 1$, $\|u_2\| = 2$, $\|u_3\| = 3$, and $\|u_4\| = 1$.
    (a) What is the dimension of the subspace $\mathcal{U}$?
    (b) Find $\|v\|$ for $v = 3u_1 - 2u_2 + 4u_3 - u_4$?
11. The vectors u, v, and w are in $\mathbb{R}^n$ and we are given that $\|u\| = 1$, $\|v\| = 2$, $\|w\| = 3$, that $\langle u, v\rangle = -1$, $\langle u, w\rangle = 2$, and that w is perpendicular to v.
    (a) Find $\|u - 3v + 2w\|$.
    (b) Show that u, v, and w are linearly independent.
12. (a) Show that $v_1 = (1, 1, 1, 1)$; $v_2 = (1, 1, -1, -1)$; $v_3 = (1, -1, 1, -1)$; and $v_4 = (1, -1, -1, 1)$ form an orthogonal basis for $\mathbb{R}^4$.
    (b) Write $w = (2, 1, -1, 2)$ as a linear combination of v_1, v_2, v_3, and v_4.
13. (a) Show that $v_1 = (1, -1, 1)$ and $v_2 = (3, 2, -1)$ are orthogonal vectors in $\mathbb{R}^3$.
    (b) Is $w = (2, 1, -1)$ in the subspace spanned by v_1 and v_2?
    (c) Find a non-zero vector in $\mathbb{R}^3$ that is perpendicular to each of v_1 and v_2.
14. (a) Show that $v_1 = (1, 0, 1, 1)$; $v_2 = (1, 1, -1, 0)$; and $v_3 = (1, -1, 0, -1)$ are orthogonal vectors in $\mathbb{R}^4$.
    (b) Is $w = (6, -1, 2, 1)$ in the subspace spanned by v_1, v_2, and v_3?
    (c) Find a non-zero vector in $\mathbb{R}^4$ that is perpendicular to each of v_1, v_2, and v_3.