
Homework 1

1. Let $u = (1, -1, 3)$, $v = (0, 2, -1)$, and $w = (3, 1, 1)$. Evaluate the following expressions:

(a) $4u$ (b) $-3v$ (c) $u + w$ (d) $4u - 3v$ (e) $2u - 4v + 3w$

2. Let $u = (2, 1, 0, -3)$, $v = (1, 0, 3, -1)$, and $w = (2, 0, 6, -2)$. Evaluate the following expressions:

(a) $3u - 2v$ (b) $2u + v - 3w$ (c) $\alpha u + \beta v + \gamma w$

3. Let u , v , and w be vectors as in the previous problem.

- (a) Find α and β , if possible, so that $\alpha u + \beta v = (1, 2, -9, -3)$.
 (b) Find α and β , if possible, so that $\alpha u + \beta w = (3, -1, 2, 0)$.
 (c) Find α and β , if possible, so that $\alpha v + \beta w = (-1, 0, -3, 1)$.

4. Let $M = \begin{pmatrix} 0 & 2 & -1 \\ 3 & 1 & 1 \end{pmatrix}$ and $N = \begin{pmatrix} -1 & 0 & 4 \\ 1 & 1 & -2 \end{pmatrix}$. Evaluate the following expressions.

(a) $3M - 2N$ (b) M' (c) $3M' - 2N'$ (d) $(3M - 2N)'$

5. Show that if E is square, then $E + E'$ is Hermitian.

6. Let $M = \begin{pmatrix} 0 & 2 & -1 \\ 3 & 1 & 1 \end{pmatrix}$ and $N = \begin{pmatrix} -1 & 0 & 4 \\ 1 & 1 & -2 \end{pmatrix}$.

Evaluate the following expressions.

(a) MM' (b) $M'M$ (c) MN' (d) $N'M$

7. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 1 & 1 \\ -1 & 0 & 3 \end{pmatrix}$, $C = \begin{pmatrix} 1 & -3 \\ -1 & 3 \\ -1 & 3 \end{pmatrix}$,

$D = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$, and $E = \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}$. Evaluate the following expressions when possible; say

Undefined when the arithmetic in the expression cannot be carried out.

(a) $3A - 2B$ (b) AE (c) AB (d) AC (e) CA
 (f) EA (g) $E'A$ (h) $AB' + D$ (i) A^2 (j) D^2

8. Let $A = \begin{pmatrix} 5 & -4 & 1 \\ 12 & -11 & 6 \\ 10 & -10 & 8 \end{pmatrix}$

and $u = (1, -1, 2)$, $v = (1, 1, 0)$, $w = (1, 2, 1)$, $e_1 = (1, 0, 0)$, and $e_2 = (0, 1, 0)$.

- (a) Find Au .
 (b) Find Av .
 (c) Find Aw .
 (d) Find Ae_1 and Ae_2 . Let $e_3 = (0, 0, 1)$; guess what Ae_3 is, then compute it.

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9. Let $S = \begin{pmatrix} 0 & 2 & -1 \\ 2 & 1 & 3 \\ -1 & 3 & -2 \end{pmatrix}$.
- Find S' .
 - What special property does S have?
 - What is $S + I$. How do you know which identity matrix to add to S ?
 - Find S^2 and S^3 .
 - Show that if T is any Hermitian matrix, then T^2 is Hermitian also.
10. Let $P = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, $Q = \begin{pmatrix} 2 & -1 & 1 \\ -3 & 4 & -2 \\ 5 & 3 & -5 \end{pmatrix}$, and let D be the diagonal matrix $D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.
- Find DP and DQ .
 - If E is the diagonal matrix with diagonal entries α , β , and γ , and R is a matrix, describe ER .
 - Find PD and QD .
 - If E is the diagonal matrix with diagonal entries α , β , and γ , and R is a matrix, describe RE .
11. Let $S = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$ and let $T = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -1 & -2 \end{pmatrix}$.
- Let $C_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, let $C_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, and let $C_3 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.
- Find SC_1 , SC_2 , and SC_3 .
 - Find ST and compare your answer with the results of part a).
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