Homework Supplement 2

From the textbook: page 279: 3, 4, 5, 6, 10, 15   page 308: 1, 4, 12

1. Let $\mathbf{Q}$ denote the set of rational numbers.

    (a) Explain why it is possible to write $\mathbf{Q} = \{q_1, q_2, q_3, \cdots\}$ where q_1 is the 'first' rational number, q_2 is the 'second' rational number, etc.

    (b) Show that $\mathbf{Q} \times \mathbf{Q}$ is countable.

2. The sequence a_n is defined recursively by $a_1 = 1$ and $a_{n+1} = 1 + \dfrac{1}{2 + a_n}$.

    (a) Write the first 4 terms of the sequence.

    (b) Show that for each positive integer n, we have $a_n \geq 1$.

    (c) Use your answer to part (b) to show that for each positive integer n, we have $a_n \leq 4/3$.