

Although in most of the course we will be dealing with real numbers, when we get to eigenvalues, we will be forced to consider complex numbers. When complex numbers arise, I will use them without apology. If you are unfamiliar with complex numbers, you may wish to learn a little about them. Any source is acceptable; some possibilities are high school algebra texts, “engineering math” texts, complex analysis texts (for example, Churchill’s *Complex Variables* is on reserve in the Math library), there are even a few pages in Strang (pages 217-219). The ability to solve the following problems indicates adequate knowledge for this course. These problems are for practice only, they will not be handed in.

Practice Problems

1. Let $z = 4 - 5i$.

Find: (a) $\operatorname{Re}(z)$ (b) $\operatorname{Im}(z)$ (c) $|z|$ (d) \bar{z} .

2. Compute:

$$\begin{array}{ll} \text{(a)} (3 + 2i)(2 - i) + i(-2 + i) & \text{(b)} (2 - 3i)^2(4 + 2i) \\ \text{(c)} (2 - i)^2 + (1 + 3i)^2 & \text{(d)} \left(\overline{(2 - i)}\right)^2 + \left(\overline{(1 + 3i)}\right)^2 \text{ see (c)} \\ \text{(e)} \frac{1}{3 + 4i} & \text{(f)} \frac{4 - 2i}{1 + i} \\ \text{(g)} \frac{2 + 3i}{(2 - i)^2} + \frac{i}{1 + i} & \text{(h)} \left| \frac{1 + 3i}{2 - i} \right|. \end{array}$$

3. Find all (3) roots of the equation

$$z^3 - 3z^2 + 7z - 5 = 0.$$

Answers:

1. (a) 4 (b) -5 (*not* $-5i$!) (c) $\sqrt{41}$ (d) $4 + 5i$.

2. (a) $7 - i$ (b) $4 - 58i$ (c) $-5 + 2i$ (d) $-5 - 2i$ [*i.e. the conjugate of (c)*]
 (e) $\frac{3}{25} - \frac{4}{25}i$ (f) $1 - 3i$ (g) $.26 + 1.18i$ (h) $\frac{|1+3i|}{|2-i|} = \frac{\sqrt{10}}{\sqrt{5}} = \sqrt{2}$

3. The roots are 1, $1 + 2i$, and $1 - 2i$.