Math 204-2, Fall 2008
Proof Writing Assignment
FIRST DRAFT DUE: Friday, September 19

Please hand in solutions to all problems. Your solutions should be written as carefully and clearly as possible. Your grade will largely be based on the clarity of your exposition.

If you use a theorem from class or from the book, please *state it clearly* (although you don't need to reprove it).

A) Let A be an $m \times n$ matrix with $m \neq n$. Show that A cannot have both a left and a right inverse (in other words, there cannot exist $n \times m$ matrices B and C such that $AB = I_m$ and $CA = I_n$).

This problem is very closely related problem 21 from Section 1.8. Please give a complete solution, however: do not just refer to problem 21.

B) A *directed graph* G is a collection of nodes (or vertices) connected by arrows (or directed edges). Each edge represents a path connecting the nodes at its endpoints, and these edges may be travelled only in the direction specified by the arrows. Associated to any graph G with n vertices is an $n \times n$ matrix A, called the incidence matrix, in which the (i, j)th entry is 1 if there is an edge from node i to node j, and zero otherwise. See Section 1.12 for examples.

Let G be a directed graph, and let A be its incidence matrix. Prove, by induction on k, that the number of directed paths of length k connecting two nodes i and j is given by the (i, j)th entry of the matrix A^k (the product of A with itself k times). Note that this is Theorem 2 on page 162 of the text.

C) Let $A_1, A_2, \ldots, A_k$ be $n \times n$ matrices. Show that if the product $A = A_1 \cdots A_k$ is invertible, then so is each A_i.