

SYLLABUS FOR MATH 372B
SEMINAR IN ALGEBRAIC TOPOLOGY:
CHARACTERISTIC CLASSES

Spring 2009
Instructor: Daniel A. Ramras

Course Info:

Tues, Thurs 1:10 – 2:25, SC1404

Office Hours: Tues. 2:30-3:30, Wed. 3:00-4:00pm in SC 1408

Text: Characteristic Classes, by John Milnor and James Stasheff

Course Description:

This course will focus on the theory of characteristic classes for vector bundles. Characteristic classes are powerful cohomological invariants associated to bundles, and have widespread applications. In this course we will develop the general theory and explore some of the applications to geometry and K -theory. On the geometric side, the study of *Stiefel-Whitney classes* leads to non-immersion results for projective spaces. On the other hand, the Chern character, defined in terms of the *Chern classes* of complex vector bundles, provides a rational isomorphism between K -theory and cohomology.

Topics to be Covered:

- Smooth manifolds and their tangent bundles
- Vector bundles, clutching functions, and principal bundles
- Classifying spaces and the homotopy theory of bundles
- The Splitting Principle and the Projective Bundle Theorem
- Definitions and basic properties of Stiefel-Whitney and Chern classes
- The Grassmanians and their cohomology
- Applications of Stiefel-Whitney classes to manifolds
- The Thom Isomorphism Theorem and the Gysin Sequence
- Characteristic classes as obstructions
- K -theory and the Chern Character
- Connections, curvature, and Chern-Weil Theory

Possible additional topics:

- Pontryagin classes, transversality, and cobordism
- Representations and flat bundles
- Secondary characteristic classes of flat bundles

References:

Here are some additional references.

- Fibre bundles, by Dale Husemoller (on reserve in the Science Library).
- Basic bundle theory and K-cohomology invariants, by Dale Husemoller, et. al. (available on-line through Acorn).
- Geometry of characteristic classes, by Shigeyuki Morita.

Homework and Grading:

Grades in this course will be based on homework and class participation. There will be several graded homework assignments, as well as additional suggested problems. Students will be asked to present solutions to selected problems in class.