Math 191 Written Homework I

This assignment is designed to help you with the more difficult material from Exam 1. A first draft will be due Friday 10/16/09. This HW will count as a quiz grade.

1. **Choose two problems from Exam 1 on which you received less than half the points.**

   If there aren't two such problems, choose two that you found difficult. You may count each part of Problem 7 as a separate problem, although please do not choose more than one part from Problem 7.

2. **Solve the two problems you have chosen.**

   Look at the solutions handed out in class, your notes, and the book. Make sure you understand all of the steps in the problem.

3. **Write *explanations* of your solutions.**

   Imagine that your friend is struggling in the class, and you are tutoring her on how to solve these problems. What does your friend need to know? What do the words in the problem mean? What are the potential mistakes? You've chosen these problems because they are hard, so explain what makes them difficult or confusing.

   Your explanations should discuss:

   - what the problem is asking,
   - how to approach the problem,
   - what methods or theorems are being used and what they say,
   - what mistakes you made, and how to avoid them,
   - and the actual solution to the problem.

   Write clearly, using complete sentences.

4. **Length:** Each explanation should be approximately 1-2 typed pages (double spaced), including formulas or graphs. You may write out mathematical symbols by hand, although Microsoft Equation Editor and similar programs should allow you to type out most of what you need.

Sample Explanation: This is a sample explanation. Problem 3c) from Quiz 5 asked you to compute

$$\lim_{x \to 0} \frac{e^{x+1} \sin(x)}{x}.$$

Explanation: This problem asks us to compute the limit of the function $\frac{e^{x+1} \sin(x)}{x}$ as the variable x approaches zero. It is important to notice that this function is not well-defined when $x = 0$: if we plug in $x = 0$, we get

$$\frac{e^{x+1} \sin(x)}{x} = \frac{e^{0+1} \sin(0)}{0} = \frac{e \cdot 0}{0} = \frac{0}{0},$$

so the limit $\lim_{x \to 0} \frac{e^{x+1} \sin(x)}{x}$ is *indeterminate of type* $\frac{0}{0}$. A common mistake is to think that such a limit cannot exist. However, we have many methods for computing indeterminate limits. Remember that the limit of $f(x)$, as x approaches 0, does not depend on $f(0)$, but only on nearby values of f.

We will use the method "Compare with a limit you know," described in the handout "Methods for computing limits." We use this method because the function involves $\frac{\sin(x)}{x}$, and we learned in Section 3.6 that

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1. \tag{1}$$

To compare $\frac{e^{x+1} \sin(x)}{x}$ and $\frac{sin(x)}{x}$, we just write

$$\frac{e^{x+1} \sin(x)}{x} = e^{x+1} \cdot \frac{\sin(x)}{x}.$$

The Limit Laws tells us that:

$$\lim_{x \to 0} e^{x+1} \frac{\sin(x)}{x} = \lim_{x \to 0} e^{x+1} \cdot \lim_{x \to 0} \frac{\sin(x)}{x}.$$

This Limit Law only holds if both limits on the right exist. So when applying this Law, always remember that if one limit on the right does not exist, you'll need to go back and try another method.

In this case, we can compute both limits. The second limit is just (1). Since e^{x+1} is continuous, we can compute the first limit by plugging in $x = 0$:

$$\lim_{x \to 0} e^{x+1} = e^{0+1} = e^1 = e.$$

Putting the pieces together, we can conclude that

$$\lim_{x \to 0} \frac{e^{x+1} \sin(x)}{x} = \lim_{x \to 0} e^{x+1} \cdot \lim_{x \to 0} \frac{\sin(x)}{x} = e \cdot 1 = e.$$

2