Math 191 **Review for Exam 1** **Fall 2009**

1. Find the equation of a line that is perpendicular to the line $y = \dfrac{3}{2}x + 1$ and goes through the point $(-1, 4)$.

2. Sketch the graph of $f(x+3) - 2$, where $f(x) = x^2$ for $-4 \le x \le 4$.

3. Estimate the instantaneous rate of change of $f(x) = \sqrt{x+5}$ at $x = -1$ by finding the slopes of secant lines.over the intervals $[-4, -1]$ and $[-1, 4]$.

4. Find $\sin\theta, \cos\theta$ and $\csc\theta$ if $\cot\theta = 6$.

5. Find $\sin\theta, \cos\theta$ and $\sec\theta$ if $\tan\theta = -7$ and $\frac{3\pi}{2} \le \theta < 2\pi$.

6. Find $\cos\theta$ and $\tan\theta$ if $\sin\theta = 0.4$ and $\frac{\pi}{2} \le \theta < \pi$.

7. Solve for $0 \le \theta < 2\pi$: $\sqrt{2}\sin(3\theta) = \sin(6\theta)$.

8. Solve for $0 \le \theta < 2\pi$: $\sin\theta = \cos(2\theta)$.

9. Find the domain of $f(x) = \ln(\ln x)$

10. Find a domain on which f is one-to-one and a formula for the inverse of f^{-1} restricted to this domain:

    (a) $f(x) = \frac{x+3}{x+2}$.

    (b) $f(x) = \frac{3}{\sqrt{x^2+2}}$

11. Compute without using a calculator: $\sin^{-1}(\sin(-\frac{5\pi}{6}))$ and $\tan^{-1}(\tan(\frac{3\pi}{4}))$

12. Simplify by referring to the appropriate triangle or trigonometric identity:

$$\cot(\sec^{-1} x).$$

13. Solve each equation for x:

    (a) $e^{2x+1} = 4e^{5-3x}$.

    (b) $\ln x + \ln(x-1) = 1$.

14. Calculate directly, without using a calculator: $\log_5 10 + \log_5 20 - 3\log_5 2$.

15. If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is $f(t) = 1000 \cdot 2^{t/3}$. Find the inverse of this function and explain its meaning. When will the population reach $50,000$?

16. Do problems 38 and 46 on p. 78 (section 2.2).

17. Evaluate $\lim_{x \to 3} (x^2 + 9x^{-3})$.

18. Assume that $\lim_{x \to -2} f(x) = 3$, compute

    (a) $\lim_{x \to -2} x^2 f(x)$.

    (b) $\lim_{x \to -2} (x+1)[f(x)]^2$.

19. Calculate $\lim_{x \to \pi/4} e^{\sin(x)}$.

20. Say $f(x)$ is defined by

$$f(x) = \begin{cases} x^2 + 4x & \text{for } x < 0 \\ \sin(x) & \text{for } 0 \leqslant x \leqslant \pi/2 \\ \cos^2(x) & \text{for } x > \pi/2 \end{cases}$$

At what points is $f(x)$ discontinuous? For each of these points, is the discontinuity a jump discontinuity, infinite discontinuity, removable discontinuity, or none of these?

21. Draw the graph of a function $f(x)$ with all of the following properties:

    (a) $f(x)$ has a removable discontinuity at $x = -1$,

    (b) $f(x)$ has a discontinuity at $x = 1$ which is neither a jump discontinuity, a removable discontinuity, nor an infinite discontinuity.

    (c) $f(x)$ is defined and continuous except at $x = -1$ and $x = 1$.

22. Evaluate the following limits, or explain why they do not exist.

    (a) $\lim_{x \to 5} \dfrac{x^2 - 11x + 30}{x - 5}$

    (b) $\lim_{h \to 0^+} \dfrac{\sqrt{5h + 10h^2} - \sqrt{5h}}{h}$

    (c) $\lim_{x \to 2} \sin\left(\dfrac{x - 2}{x^2 - 4x + 4}\right)$

    (d) $\lim_{x \to 9} \left[\dfrac{1}{\sqrt{x} - 3} - \dfrac{6}{x - 9}\right]$

23. Find $\lim_{x \to 0} x^2 \cos\left(\dfrac{1}{x}\right)$.

24. Show that $\cos x = x$ has a solution on the interval $[0, 1]$. Justify your answer.

25. Use the definition of derivative to find $f'(2)$ for $f(x) = \sqrt{3 + x}$.