

Math 191, Fall 2009
Common Arithmetic Mistakes
(and how to avoid them)

This sheet lists some common arithmetic mistakes that I've been seeing on quizzes.

0) **Think about what you're writing.** Many of you write things on the quizzes that I bet you know are nonsense when you look back at them. Don't apply formulas and techniques from memory, without thinking through the steps. This leads to bizarre and easily corrected mistakes. This happens a lot with:

- canceling in fractions (see 3) and 4) below), dividing by fractions (see 6) below), and anything else involving fractions!
- multiplying by the conjugate.

1) **Imaginary formulas.** There is no formula like $\sin(3t) = 3\sin(t)$. That would be nice, I guess, but it's completely nonsense. Sine is a complicated function, so don't make up simple formulas for it! Learn the formulas for exponentials and logs, and the Pythagorean formula. If you know these well, you're less likely to make up your own nonsense formulas.

2) **Don't ignore exponents.** There is a huge difference between $2^{\sin(x)}$ and $2\sin(x)$. No one in this class has confused x^3 with $x3$, so I know you can tell the difference!

3) **Be careful when cancelling from the top and bottom of fractions:** While it is true that $\frac{ab}{cb} = \frac{a}{c}$ (the b 's cancel, because you've multiplied by b and then divided by b) you need to be sure to cancel from each term on the top and each term on the bottom. Said another way, you need to factor b from top and bottom before this cancellation. So

$$\frac{x^3 \sin(x) + x^6 \sin(x)}{3 \sin(x) + x \sin(x)} = \frac{\sin(x)(x^3 + x^6)}{\sin(x)(3 + x)} = \frac{x^3 + x^6}{3 + x}$$

but

$$\frac{x^3 \sin(x) + x^6}{3 \sin(x) + x \sin(x)} \neq \frac{x^3 + x^6}{3 + x}$$

(To correct the second equation, you could replace x^6 by $\frac{x^6}{\sin(x)}$ on the right.)

4) **You can't cancel functions as though they were numbers.** The following equation false because $\sin(3x)$ is not the product of something called \sin with $3x$, so you can't "cancel the \sin 's":

$$\frac{\sin(3x)}{\sin(x^3 + 2)} \neq \frac{3x}{x^3 + 2}.$$

Of course, you *can* cancel functions evaluated at a point as though they were numbers, because when you evaluate a function at a point, you get a number! For example,

$$\frac{\sin(3x)}{\sin(3x) \sin(x^3 + 2)} = \frac{1}{\sin(x^3 + 2)}$$

because you can cancel $\sin(3x)$ from the top and bottom.

Similarly, you cannot cancel the arguments of functions (the values being fed into the functions) as though they were factors:

$$\frac{\sin(x^2)}{\sin(x)} \neq \frac{\sin(x)}{\sin(1)},$$

because you can't cancel the x 's on the inside of the function.

5) **Combining fractions over a common denominator.** You can avoid some common mistakes if you remember that:

$$\frac{a}{b} + \frac{c}{d} \neq \frac{a+c}{b+d},$$

$$\frac{a}{b} - \frac{c}{d} \neq \frac{a-c}{b-d}.$$

The correct formulas are very easy:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd},$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-cb}{bd}.$$

Note that you can separate out the second fraction into $\frac{ad}{bd} + \frac{cb}{bd}$, and then you see why these are the same (the b 's cancel, as in 3) above). When you combine fractions, always look at what you've written to make sure it cancels out to give back the original expression (see Mistake 0) above).

Note that this works if a, b, c , and d are numbers, but also if they're functions. So for example,

$$\frac{e^x}{\ln(x)} + \frac{x^2}{\tan(x)} = \frac{e^x \tan(x) + \ln(x)x^2}{\ln(x)\tan(x)}.$$

6) **Dividing by a fraction is the same as multiplying by the reciprocal:** When I was In 5th grade, Ms. Keefe made Katie Sauer do a head stand in front of the class in order to illustrate this. She was right: I still think of that every time I divide by a fraction.

For example,

$$\frac{3}{2/4} = 3 \cdot (4/2) = \frac{3 \cdot 4}{2}$$

and

$$\frac{\sin(x)}{x^2/e^x} = \frac{\sin(x) \cdot e^x}{x^2}.$$

If you find this confusing or surprising, just multiply both sides of the first equation by $2/4$ and you'll see why it's true.

It's important to understand the relationship between the previous examples and the following:

$$\frac{\frac{8}{2}}{2} = \frac{8}{2 \cdot 2} \tag{1}$$

and

$$\frac{\frac{\sin(x)}{x^2}}{e^x} = \frac{\sin(x)}{e^x \cdot x^2}.$$

Here we're dividing, and then dividing again, which is *different* from dividing by a fraction. In the equation (1), note that the left hand side is really just $\frac{4}{2} = 2$, since the top part is $\frac{8}{2} = 4$. Now you should see why the equation is true: on both sides, we take the number 8, divide by 2, and then divide by 2 again. If you like, you can think of equation (1) in this way: on the bottom, 2 can be thought of as the fraction $2/1$, so dividing by 2 is the same as multiplying by the reciprocal of $2/1$, namely $1/2$. So

$$\frac{\frac{8}{2}}{2} = \frac{8}{2} \cdot \frac{1}{2} = \frac{8}{2 \cdot 2}.$$

Combining these two paragraphs, you should now see why

$$\frac{\sin(x)}{\frac{x^2}{e^x}} \neq \frac{\sin(x)}{e^x}.$$

Order of operations is important.