Math 191 Review for Exam 2

Fall 2009

1 Find all the points where the tangent line to $f(x) = x^3 - 6x^2 - 15x$ is horizontal. $(x = 5, x = -1)$

2 The height of an object falling due to gravity is given by $s(t) = s_0 + v_0 t - \frac{1}{2}gt^2$. Suppose a toy rocket is fired from the ground with an initial velocity of 160 ft/sec.

(a) Determine when the rocket reaches its maximum height and the maximum height. (5 sec, 400 ft)

(b) When does the rocket hit the ground? What is its velocity at that time? (10 sec, -160 ft/sec^2)

3 Let $f(x) = e^x \sin x$. Calculate f' and f''. $(e^x(\sin x + \cos x), 2e^x \cos x)$

4 Use implicit differentiation to calculate $\dfrac{dy}{dx}$: $xy^2 = 1 - \cos(x + y)$. $\left(\frac{\sin(x+y)-y^2}{2xy-\sin(x+y)}\right)$

5 For the ellipse: $2x^2 + 3y^2 + 2xy = 15$

(a) Write an equation for the tangent line to the ellipse at (2,1). $(y = -x + 3)$

(b) Find all points on the ellipse at which the tangent line is horizontal. $\left(\left(\frac{\sqrt{6}}{2}, -\sqrt{6}\right), \left(-\frac{\sqrt{6}}{2}, \sqrt{6}\right)\right)$

6 Find the derivatives of the following functions.

(a) $y = \cos^{-1}(t) - \sqrt{1-t^2}$. $\left(\dfrac{t-1}{\sqrt{1-t^2}}\right)$

(b) $y = e^{\tan^{-1} x^2}$. $\left(\dfrac{2x\,e^{\tan^{-1} x^2}}{1+x^4}\right)$

(c) $y = x \sin^{-1}(e^x)$. $\left(\sin^{-1}(e^x) + \frac{xe^x}{\sqrt{1-(e^x)^2}}\right)$

(d) $g(s) = s\cos(e^{3s-7})$. $(\cos(e^{3s-7}) - 3\,s\,\sin(e^{3s-7})\,e^{3s-7})$

(e) $h(x) = \tan^2(xe^x)$. $(2\,\tan(xe^x)\,\sec^2(xe^x)\,(e^x + xe^x))$

(f) $y = \dfrac{3 + x^{-3}}{\ln x + 1}$. $\left(-\frac{4+3\ln x+3x^3}{x^4(\ln x+1)^2}\right)$

7 Find an equation of the tangent line at the point $x = 1$ to $y = x^{(3^x)}$. $(y = 3x - 2)$

8 Use logarithmic differentiation to find the derivative of $y = \sqrt{\frac{x(x+5)}{(3x+7)(5x-1)}}$.
$\left(\frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{x(x+5)}{(3x+7)(5x-1)}}\left(\frac{1}{x} + \frac{1}{x+5} - \frac{3}{3x+7} - \frac{5}{5x-1}\right)\right)$

9 If a snowball is melting so that its surface area is decreasing at a rate of 0.5 cm^2/min, find the rate at which the radius is decreasing when the radius is 4 cm. $(-\frac{1}{64\pi}$ cm/min)

10 As a man walks away from a 12-ft lamppost, the tip of his shadow moves twice as fast as he does. What is the man's height? (6 ft)

11 Solve problem 17, p. 205.

12 Estimate Δf using Linear Approximation for $f(x) = \cos x$ at $a = \pi/4$ where $\Delta x = 0.1$. $(-\frac{\sqrt{2}}{20})$

13 Find the linearization of $f(x) = \sin^{-1} x$ at $a = 1/2$. $\left(\frac{\pi}{6} + \frac{2}{\sqrt{3}}(x - \frac{1}{2})\right)$

14 Find the maximum and minimum values of the following functions on the given interval.

(a) $f(x) = x^3 - 3x + 1$, $[0, 2]$. $(f(2) = 3, f(1) = -1)$

(b) $f(x) = x - \sin x$, $[0, 2\pi]$. $(f(2\pi) = 2\pi, f(0) = 0)$

(c) $f(x) = xe^{-x}$, $[0, 2]$. $(f(1) = 1/e, f(0) = 0)$

(d) $f(x) = x^5 - 3x^2$, $[-1, 5]$. $((f(5) = 3050, f(-1) = -4)$

15 Find the critical points of $f(t) = 4t - \sqrt{t^2 + 1}$. (no critical points)

16 Estimate $\sin(\pi/4 + \pi/16)$ using the linearization of $f(x) = \sin x$ at $a = \pi/4$. $\left(\frac{\sqrt{2}}{2}(1 + \frac{\pi}{16})\right)$