

1. Compute the following limits:

$$(a) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x}{\csc x} \quad (b) \lim_{x \rightarrow 0^+} \left( \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x^2 + x}} \right) \quad (c) \lim_{x \rightarrow 4} \cos^{-1} \left( \frac{x}{8} \right) \quad (d) \lim_{x \rightarrow 0} \frac{\tan x}{x}$$

$$(e) \lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x} \quad (f) \lim_{x \rightarrow \infty} x^{1/x} \quad (g) \lim_{x \rightarrow 1} \frac{e^x - e}{\ln x}.$$

2. For the function  $f(x) = \begin{cases} x^2 + c, & x < 3 \\ 2x - c, & x \geq 3 \end{cases}$ , determine the value of  $c$  that makes the function continuous.

3. Use the **definition** of derivative to find  $f'(x)$  for  $f(x) = \frac{1}{\sqrt{x+3}}$ .

4. Find the derivative of the following functions:

$$(a) f(x) = x^2 e^{-x^2} \quad (b) f(x) = e^{-3x} \sin 2x \quad (c) g(x) = (2x+1)^4 (x^3 - x + 2)^5$$

$$(d) f(x) = \sin^{-1}(2x+1) \quad (e) f(x) = x^{\cos^{-1} x} \quad (f) g(x) = e^{\tan^{-1} x}$$

$$(g) f(x) = \log_4(x^2 - 5) \quad (h) f(x) = x \ln(e^x + 1) \quad (j) f(x) = 7^{1-x^3}$$

5. Find all points on the graph of  $y = \frac{\cos(x)}{2 + \sin(x)}$  at which the tangent line is horizontal.

6. Find an equation of the tangent line to the curve  $y = \frac{x}{1+x^2}$  at  $x = 2$ .

7. Calculate the first and second derivatives of each of the following functions. It is not necessary to simplify your answers.

$$(a) f(x) = e^x \cos(x)$$

$$(b) g(x) = x^{-3/4} - 4x^7 + x^{1/2}$$

$$(c) h(x) = x^3 \sin(x)$$

$$(d) k(x) = (x^2 + x) \tan(x)$$

8. Use implicit differentiation to find  $\frac{dy}{dx}$ :

$$(a) x^2 - 2xy + y^3 = 3$$

$$(b) y \sin(x^2) = x \sin(y^2)$$

9. Use logarithmic differentiation to find the derivative of the following functions:

$$(a) y = \frac{\sqrt{x+5} \cdot (x-2)^3}{e^{x^2} (x^3+1)^4}$$

$$(b) y = (\sin x)^x$$

10. A plane is flying horizontally at an altitude of 2 miles and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 4 miles away from the station.

11. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8m from the dock?

12. Find the linearization at  $x = 1$  of  $y = e^x \ln x$ .

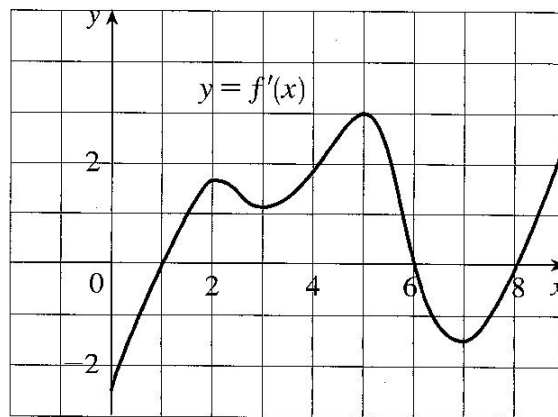
13. Find the maximum and minimum values of the function on the given interval:

$$f(x) = \frac{1-x}{x^2+3x}, \quad [1, 4]$$

14. Find the critical points and the intervals on which the function is increasing or decreasing, and apply the First Derivative Test to each critical point to determine whether it is a local maximum, a local minimum, or neither. Additionally, find the intervals on which the function is concave up or down, and any point of inflection:

$$y = \theta + \cos \theta, \quad [0, 2\pi].$$

15. The graph of the derivative  $f'$  of a continuous function  $f$  is shown below:



- (a) On what intervals is  $f$  increasing or decreasing?
- (b) At what values of  $x$  does  $f$  have a local maximum or minimum?
- (c) On what intervals is  $f$  concave upward or downward?
- (d) State the  $x$ -coordinates of the points of inflection.
- (e) Assuming that  $f(0) = 0$ , sketch a possible graph of  $f$ .

16. Sketch the graph of the following functions. Indicate the asymptotes, the local extrema and points of inflection:

(a)  $f(x) = x - 2\ln(x^2 + 1)$

(b)  $f(x) = \frac{x}{\sqrt{x^2 + 1}}$

17. Problem 8, p. 265.

18. Problem 10, p. 265.

19. Consider the graph of the function  $f(x) = \frac{x^3}{1+x^4}$ .

a) Find a formula for the slope of the secant line between  $(x, f(x))$  and  $(-x, f(-x))$ .

b) At what point  $x$  is the slope in part a) maximized? Make sure to explain why this point is a global maximum.

20. Problem 31, p. 267.